2020B F=ma Exam: Problem 5

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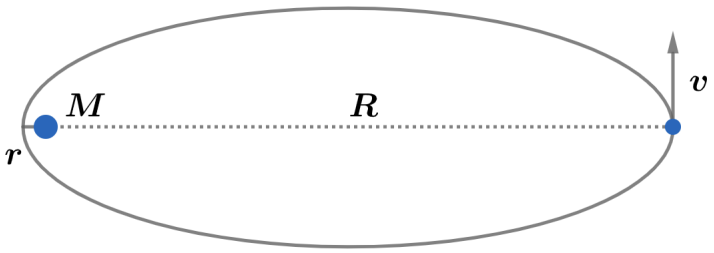

The angular momentum of the system is given by

$$L = mvR$$

Recall the angular momentum of an elliptical orbit is given by

$$L = mb\sqrt{\frac{GM}{a}}$$

By definition for an ellipse, we have

$$rR = (a-c)(a+c) = a^2 - c^2 = b^2$$

so

$$mvR = m\sqrt{rR}\sqrt{\frac{GM}{a}}$$

$$v^2R^2 = rR\frac{GM}{a}$$

Since v is small, $R/a \approx 2$ so

$$2GMr = v^2R^2$$

$$r = \frac{R^2v^2}{2GM}$$

and the answer is $\boxed{\text{C}}$.