2020A F=ma Exam: Problem 22

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We have momentum conservation so $\Delta\vec{P} = 0$ in any inertial frame. We have

$$K_i = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$

$$K_f = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

so

$$\Delta K = \frac{1}{2}m_1(v_{1f}^2 - v_1^2) + \frac{1}{2}m_2(v_{2f}^2 - v_2^2) = \frac{1}{2}m_1(\vec{v}_{1f} - \vec{v}_1)\cdot(\vec{v}_{1f} + \vec{v}_1) + \frac{1}{2}m_2(\vec{v}_{2f} - \vec{v}_2)\cdot(\vec{v}_{2f} + \vec{v}_2)$$

$$= \frac{1}{2}\Delta\vec{p}_1\cdot(\vec{v}_{1f} + \vec{v}_1) + \frac{1}{2}\Delta\vec{p}_2\cdot(\vec{v}_{2f} + \vec{v}_2) = \frac{1}{2}\Delta\vec{p}_1\cdot(\vec{v}_{1f} + \vec{v}_1 - \vec{v}_{2f} - \vec{v}_2)$$

where we used the fact that $\Delta\vec{p}_1 = -\Delta\vec{p}_2$ by momentum conservation. We see that ΔK remains invariant under boosts of any velocity $\vec{u}$ i.e. adding $\vec{u}$ to all the velocity vectors.

Thus, $\boxed{\Delta\vec{P} \text{ and } \Delta K \text{ are independent of inertial frame}}$.