

# 2007 F=ma Exam: Problem 11

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Recall for rotational motion,

$$\begin{aligned}\tau &= I\alpha \\ \alpha &= \frac{\tau}{I}\end{aligned}$$

From kinematics,

$$\omega = \omega_0 + \alpha t = \frac{\tau t}{I}$$

Thus, the kinetic energy of the objects after time  $t$  is given by

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}I\left(\frac{\tau t}{I}\right)^2 = \frac{\tau^2 t^2}{2I}$$

The torques on all the objects are the same so

$$K \propto \frac{1}{I}$$

Recall the moments of inertia:

$$I_{disk} = \frac{1}{2}MR^2$$

$$I_{hoop} = MR^2$$

$$I_{sphere} = \frac{2}{5}MR^2$$

Hence,

$$\boxed{K_{hoop} < K_{disk} < K_{sphere}}$$