Exercise 4.21

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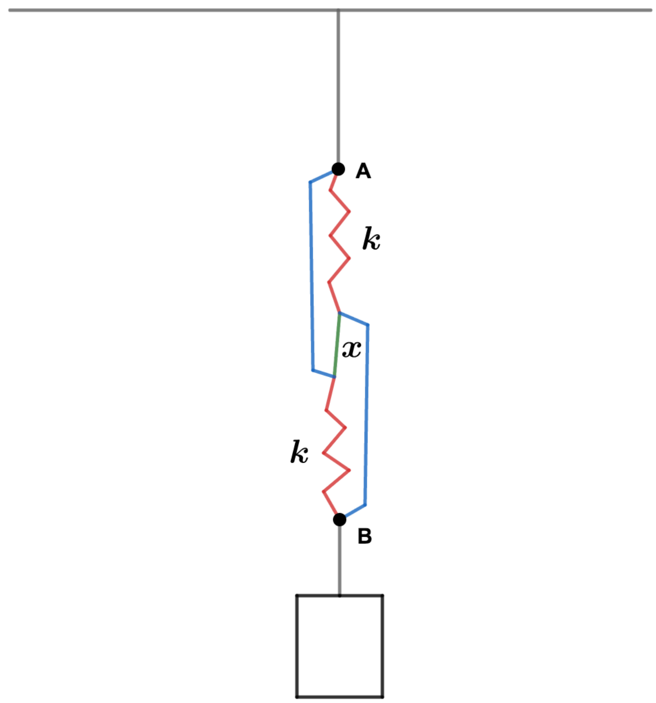


Consider the configuration shown. Initially, the blue strings are relaxed and the green string of length x is taut. The springs are in series so their effective spring constant is $k_e = k/2$. The effective spring extension is

$$\Delta L_0 = \frac{F}{k_e} = \frac{mg}{k/2} = \frac{2mg}{k}$$

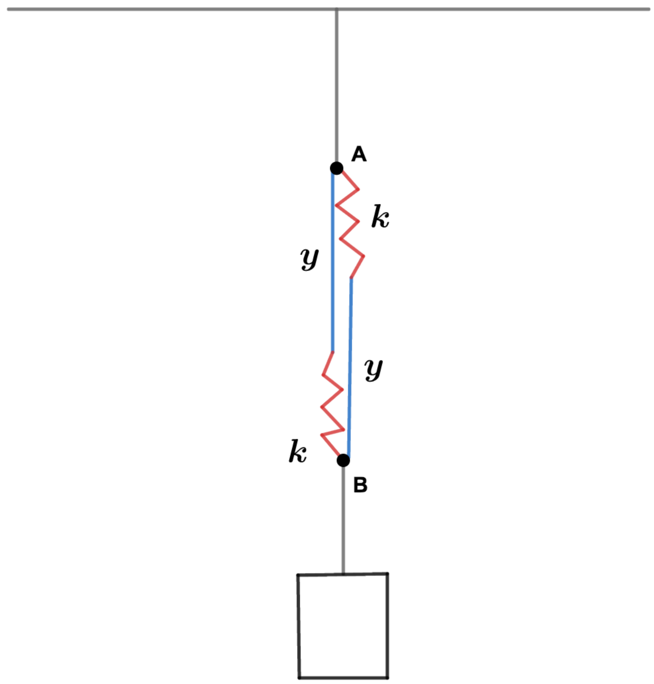
If the springs have resting length L, the distance AB is then

$$d_0 = L + x + L + \Delta L_0 = 2L + x + \frac{2mg}{k}$$

Take the length of the blue strings to be

$$y = \frac{d_0 - x}{2} + x + \delta = L + x + \frac{mg}{k} + \delta$$

such that they each have a small δ length of string in excess. Now cut the green string.



The springs are in parallel so their effective spring constant is $k_e = 2k$. The effective spring extension is

$$\Delta L_f = \frac{F}{k_e} = \frac{mg}{2k}$$

The distance AB is now

$$d_f = L + y + \Delta L_f = 2L + x + \frac{3mg}{2k} + \delta$$

We have

$$d_0 - d_f = \left(2L + x + \frac{2mg}{k}\right) - \left(2L + x + \frac{3mg}{2k} + \delta\right) = \frac{mg}{2k} - \delta$$

Thus, as long as

$$\delta < \frac{mg}{2k}$$

the mass will subsequently rise up.