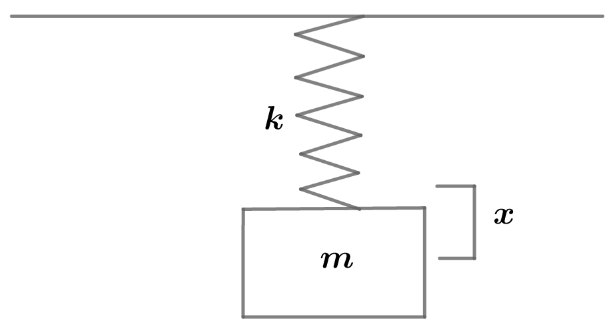
Vertical spring

Kevin S. Huang

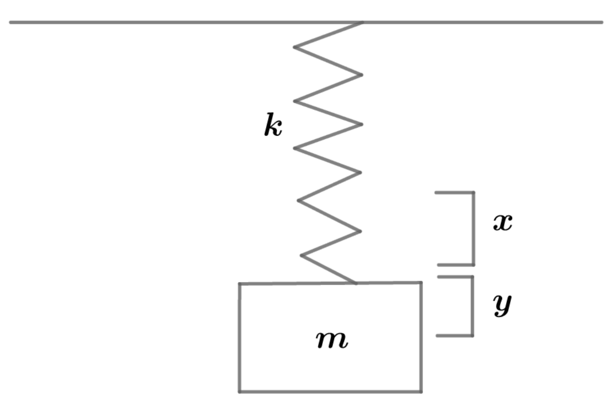
Consider a vertical spring:



The equilibrium is shifted down by x with

$$-kx + mg = 0$$
$$x = \frac{mg}{k}$$

Suppose the mass is displaced by y from the new equilibrium.



Then the restoring force is

$$F = -k(x + y) + mg = -ky - kx + mg$$
$$\boxed{F = -ky}$$

The potential energy gained by the system is

$$U = U_f - U_i$$

$$U_f = \frac{1}{2}k(x+y)^2 - mg(x+y)$$

$$U_i = \frac{1}{2}kx^2 - mgx$$

$$U = \frac{1}{2}k(x+y)^2 - mg(x+y) - \frac{1}{2}kx^2 + mgx$$

Simplifying,

$$U = \frac{1}{2}k(2xy + y^2) - mgy = \frac{1}{2}ky^2 + (kx - mg)y$$

$$\boxed{U = \frac{1}{2}ky^2}$$

Hence, a vertical spring is equivalent to a horizontal spring with a shifted equilibrium. In particular, note that the gravitational potential energy of the mass is accounted for by the equivalent spring.