Exercise 3.65

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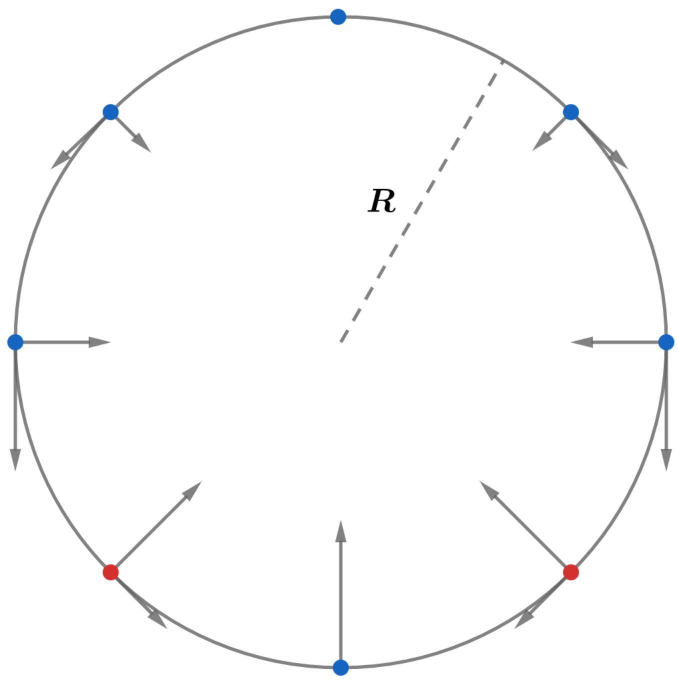
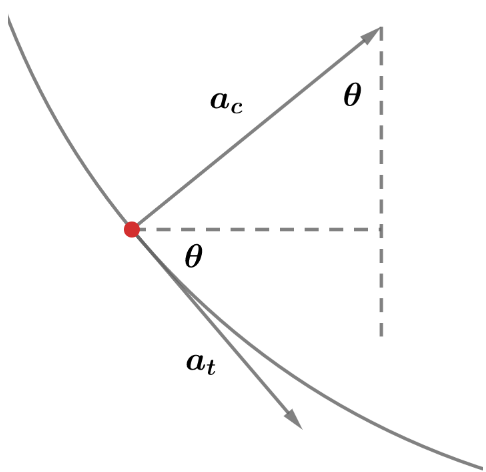


By analyzing the tangential and radial components of acceleration, we see that the acceleration can only be horizontal at two points on either side of the bottom, making angle θ to the vertical.



We require zero acceleration in the vertical direction:

$$a_c \cos\theta = a_t \sin\theta$$

with

$$a_c = \frac{v^2}{R}$$
$$a_t = g\sin\theta$$

Using the fact that the bead's speed after it has fallen height h is given by $v = \sqrt{2gh}$,

$$h = R(1+\cos\theta)$$
$$v = \sqrt{2gR(1+\cos\theta)}$$
$$a_c = \frac{2gR(1+\cos\theta)}{R}$$

Thus,

$$2g(1+\cos\theta)\cos\theta = g\sin^2\theta$$
$$2\cos\theta + 2\cos^2\theta = \sin^2\theta$$
$$3\cos^2\theta + 2\cos\theta - 1 = 0$$
$$(3\cos\theta - 1)(\cos\theta + 1) = 0$$

Note that $\cos\theta = -1$ corresponds to the top of the hoop where there is no acceleration.

$$\cos\theta = \frac{1}{3}$$
$$\boxed{\theta = \arccos\left(\frac{1}{3}\right) \approx 70.5^\circ}$$