2018B F=ma Exam: Problem 17

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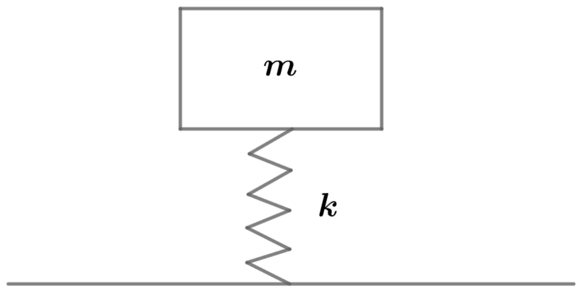

The initial energy of the system is

$$E_i = \frac{1}{2}mv^2 + mgL$$

where L is the original length of the spring. When the spring is compressed by x,

$$E = \frac{1}{2}mv'^2 + mg(L-x) + \frac{1}{2}kx^2$$

By conservation of energy,

$$\frac{1}{2}mv'^2 + mg(L-x) + \frac{1}{2}kx^2 = \frac{1}{2}mv^2 + mgL$$

$$\frac{1}{2}mv'^2 - mgx + \frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

$$v'^2 = v^2 + 2gx - \frac{k}{m}x^2$$

To maximize v',

$$\frac{dv'^2}{dx} = 2g - \frac{2k}{m}x = 0$$

$$x = \frac{mg}{k}$$

Thus,

$$v'^2 = v^2 + 2g\left(\frac{mg}{k}\right) - \frac{k}{m}\left(\frac{mg}{k}\right)^2$$

$$v' = \sqrt{v^2 + \frac{mg^2}{k}}$$

Alternatively, note the effect of a vertical spring is to shift the equilibrium by $x = mg/k$. The initial energy of the system is then

$$E_i = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

To maximize v',

$$E = \frac{1}{2}mv'^2$$

By conservation of energy,

$$\begin{aligned}
\frac{1}{2}mv'^2 &= \frac{1}{2}mv^2 + \frac{1}{2}k\left(\frac{mg}{k}\right)^2 \\
v'^2 &= v^2 + \frac{mg^2}{k} \\
v' &= \sqrt{v^2 + \frac{mg^2}{k}}
\end{aligned}$$

so the answer is $\boxed{\text{E}}$.