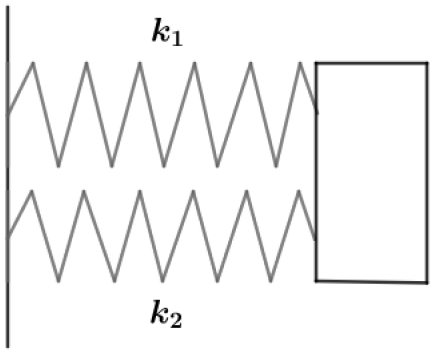
Springs in parallel and in series

Kevin S. Huang

Consider two springs in parallel:



Suppose the block is displaced Δx from equilibrium. Then we have $\Delta x_1 = \Delta x_2 = \Delta x$. The restoring force on the block is

$$|F| = k_1 \Delta x_1 + k_2 \Delta x_2 = (k_1 + k_2)\Delta x$$

Thus, the effective spring constant is

$$k = \frac{|F|}{\Delta x} = k_1 + k_2$$

Therefore, spring constants add in parallel.

Consider two springs in series:

Suppose the block is displaced Δx from equilibrium. Then we have $\Delta x = \Delta x_1 + \Delta x_2$. The restoring force on the block is

$$|F| = k_2 \Delta x_2$$

The net force on the connection point between the springs is zero so

$$k_1 \Delta x_1 = k_2 \Delta x_2$$
$$\frac{\Delta x_1}{\Delta x_2} = \frac{k_2}{k_1}$$

Thus, the effective spring constant is

$$k = \frac{|F|}{\Delta x} = \frac{k_2 \Delta x_2}{\Delta x_1 + \Delta x_2} = \frac{k_2}{k_2/k_1 + 1} = \frac{k_1 k_2}{k_1 + k_2}$$
$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

Therefore, the reciprocal of spring constants add in series.