2017 F=ma Exam: Problems 7-8

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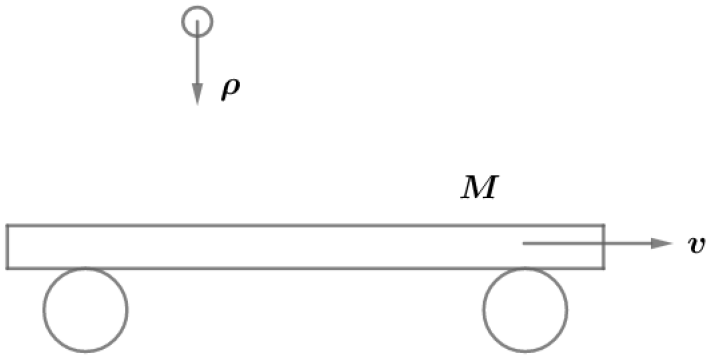


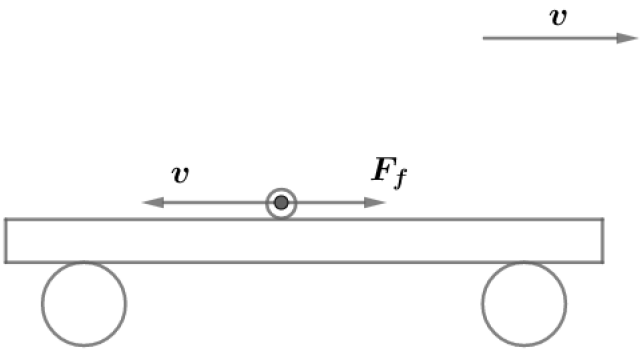
7. The kinetic energy of the train and snow is given by

$$K = \frac{1}{2}(M_t + m_s)v^2$$

Since M_t and v are constant, we have

$$\boxed{\frac{dK}{dt} = \frac{1}{2}\frac{dm_s}{dt}v^2 = \frac{1}{2}\rho v^2}$$

8. In addition to the increase in kinetic energy, we need to account for dissipation between the snow and the train. Let's move to the train frame moving at v to the right.



In this frame, the snow lands moving at v to the left and is slowed to rest. The energy lost to friction for a small piece of snow dm is

$$dE_f = \frac{1}{2}(dm)v^2 = \frac{1}{2}(\rho\, dt)v^2$$

Thus, the rate of dissipation is

$$\frac{dE_f}{dt} = \frac{1}{2}\rho v^2$$

Therefore, the power required from the engine is

$$\boxed{P = \frac{dE}{dt} = \frac{dK}{dt} + \frac{dE_f}{dt} = \rho v^2}$$