Exercise 3.47

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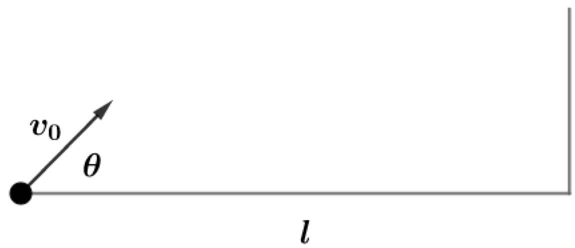

Assume $l < v_0^2/g$ since the range of a projectile is given by $R = v_0^2 \sin 2\theta/g$ so $R_{max} = v_0^2/g$.

From kinematics, the motion of the ball is given by:

$$x = v\cos\theta \cdot t$$

$$y = v\sin\theta \cdot t - \frac{1}{2}gt^2$$

Eliminating t, we obtain the trajectory of the particle:

$$y = x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta}$$

Thus, the height of the ball after traveling distance l is

$$h = l\tan\theta - \frac{gl^2}{2v^2\cos^2\theta}$$

To maximize h, we have

$$\frac{dh}{d\theta} = l\sec^2\theta - \frac{gl^2}{v^2}\tan\theta\sec^2\theta = 0$$

$$l - \frac{gl^2}{v^2}\tan\theta = 0$$

$$\tan\theta = \frac{v_0^2}{gl}$$

$$\theta = \arctan\left(\frac{v_0^2}{gl}\right)$$