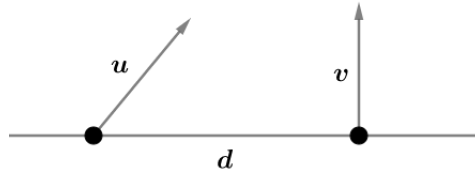


Exercise 3.45

Kevin S. Huang



If the balls collide at their highest point, they must have the same initial vertical velocity.

$$u_y = v$$

We have $v_y = 0$ at the top so for the right ball,

$$v_0 + at = v_f$$

$$v - gt = 0$$

$$t_{up} = \frac{v}{g}$$

The left ball must travel d in time t_{up} , thus

$$u_x t_{up} = d$$

$$u_x = \frac{gd}{v}$$

We have

$$u = \sqrt{u_x^2 + u_y^2} = \sqrt{\frac{g^2 d^2}{v^2} + v^2}$$

To minimize u , we have

$$\frac{d}{dv} \left(\frac{g^2 d^2}{v^2} + v^2 \right) = -\frac{2g^2 d^2}{v^3} + 2v = 0$$

$$\frac{2g^2 d^2}{v^3} = 2v$$

$$v^4 = g^2 d^2$$

$$v = \sqrt{gd}$$