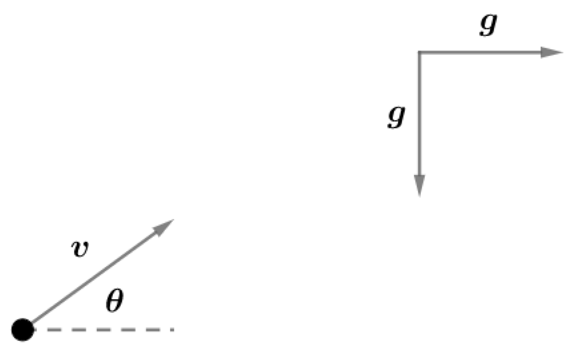
Exercise 3.42

Kevin S. Huang



At the top of the ball's trajectory, $v_y = 0$. Thus,

$$v_0 + at = v_f$$
$$v\sin\theta - gt = 0$$
$$t_{up} = \frac{v\sin\theta}{g}$$

By symmetry, $t_{down} = t_{up}$ so we have

$$T = t_{up} + t_{down} = 2t_{up} = \frac{2v\sin\theta}{g}$$

From kinematics,

$$x = v_0 t + \frac{1}{2}at^2$$

Thus, the range of the ball is

$$R = (v\cos\theta)T + \frac{1}{2}gT^2 = \frac{2v^2\sin\theta(\sin\theta + \cos\theta)}{g}$$

To maximize range, we have

$$\frac{dR}{d\theta} = \frac{2v^2}{g}(\sin 2\theta + \cos 2\theta) = 0$$
$$\tan 2\theta = -1$$
$$2\theta = \frac{3\pi}{4}$$
$$\theta = \frac{3\pi}{8} = 67.5^\circ$$