Exercise 2.36

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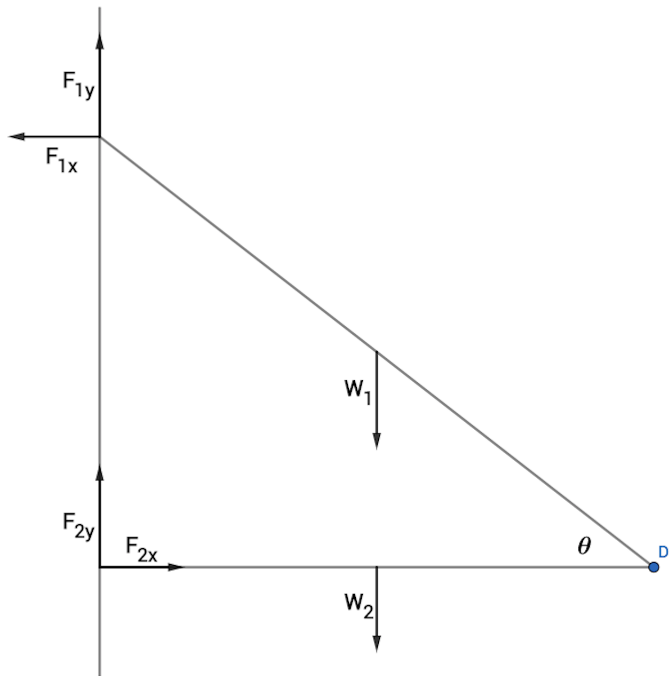

Balancing forces on the two-stick system, we have

$$F_{1x} = F_{2x}$$

$$F_{1y} + F_{2y} = W_1 + W_2$$

Balancing torques with the origin at the lower hinge,

$$F_{1x} L \tan\theta = (W_1 + W_2)\frac{L}{2}$$

We can obtain our last equation by balancing torques on the lower rod and choosing the origin at D. Then the force at the hinge has no contribution and we have

$$F_{2y} L - W_2 \frac{L}{2} = 0$$

$$F_{2y} = \frac{W_2}{2}$$

We also have

$$W_1 = \frac{\rho L g}{\cos\theta}$$
$$W_2 = \rho L g$$

Thus

$$F_{1x} = \left(\frac{\rho L g}{\cos\theta} + \rho L g \right) \frac{1}{2\tan\theta} = \frac{\rho L g (1 + \cos\theta)}{2\sin\theta}$$

$$F_{1y} = W_1 + W_2 - \frac{W_2}{2} = \frac{\rho L g (2 + \cos\theta)}{2\cos\theta}$$

$$F = \sqrt{F_{1x}^2 + F_{1y}^2} = \frac{\rho L g}{2} \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta} + \frac{(2+\cos\theta)^2}{\cos^2\theta}} = \frac{\rho L g}{2} \sqrt{\frac{4 + 4\cos\theta - 2\cos^2\theta - 2\cos^3\theta}{\sin^2\theta\cos^2\theta}}$$

$$F = \frac{\rho L g}{2} \sqrt{\frac{2(2 - \cos^2\theta)}{(1 - \cos\theta)\cos^2\theta}}$$

As $\theta \to 0$, $F \to \infty$ since $(2 - \cos^2\theta) \to 1$, $(\cos^2\theta) \to 1$, and $(1 - \cos\theta) \to 0$.
As $\theta \to \pi/2$, $F \to \infty$ since $(2 - \cos^2\theta) \to 2$, $(1 - \cos\theta) \to 1$, and $(\cos^2\theta) \to 0$.