1.2 List of strategies

This section contains a list of all the problem-solving strategies I can think of. *The list is long, so there is certainly no need to memorize it.* It would be a step backward if you spent your time worrying about covering all of the strategies, when you should instead be thinking about actually solving a problem. The best way to use this list is to read through it now, and then occasionally refer back to it, especially if you get stuck.

You will inevitably apply many of the strategies without even trying to. But others in the list might seem like meaningless gibberish for now; we're not applying them to any problems here, so there isn't much context. However, if you refer back to the list when solving problems, a given strategy will mean much more if it helps you solve a problem.

Different people think differently, of course. Some strategies might work for you, while others might not. In the end, there's no overall magic bullet for solving all problems. It just comes down to practice and doing lots of problems. But the strategies listed below should help make the practice more efficient. We've divided them into five categories: (1) Getting started, (2) Solving the problem, (3) Troubleshooting, (4) Finishing up, and (5) Looking ahead.

1.2.1 Getting started

The following nine strategies will help you get started on a problem. They don't require too much thinking; they're standard mechanical things that you can do on auto-pilot.

1. Read the problem slowly and carefully

There's no better way to waste time than to read a problem quickly in an effort to save time. If you miss a piece of the given information, you'll end up just spinning your wheels, trying to solve an unsolvable problem.

There is famous statement about the existence of known knowns (things that we know we know), known unknowns (things that we know we don't know), and unknown unknowns (things that we don't know we don't know). Leaving firmly aside who made the statement and why, you might wonder about the fourth permutation: the unknown knowns. What might those be? Well, one thing that certainly falls into this category is the information you miss when you read a problem too quickly. The information is certainly known, but you just don't know that you know it!

2. Identify the things you know, and the things you are trying to find

Identifying the known quantities enables you to see what you have to work with. And identifying the unknown quantities enables you to see what you're aiming for, which gives you some guidance in thinking about what physical principles you should consider (Strategy 10 below). Of course, as mentioned in Strategy 1, identifying the things you know requires reading the problem carefully!

The "knowns and unknowns" reference in Strategy 1 is relevant here too. We mentioned there that you want to avoid unknown knowns. You also want to avoid unknown unknowns.

These are things you're going to need to find, but you don't know yet that you need to look for them. It's much easier, of course, to reach a destination if you know what that destination is. So do your best to make sure you know what all of the unknowns are. Basically, try to make sure everything is known – even if it's (an) unknown!

3. Draw a picture

Draw a nice *big* picture, one where you can label everything clearly. Make a note of which quantities you know, and which quantities you're trying to find. Although a small set of mechanics problems involve doing only some math, the vast majority involve a setup that you really need to visualize in order to get anywhere. A picture makes things much more concrete.

4. Draw free-body diagrams

A special kind of picture is a free-body diagram. This is a picture where you draw all of the external forces acting on a given object. As with a general picture, make a note of the known and unknown quantities. Free-body diagrams are absolutely critical when solving problems involving forces. More precisely, they are necessary, and nearly sufficient. That is, many problems are impossible if you don't draw the free-body diagrams, and trivial if you do. Often the only thing that remains to be done after drawing the diagrams is to solve some $F = ma$ equations by doing some math. The physics is all contained in the diagrams. See the introduction to Chapter 4 for further discussion of free-body diagrams.

5. Strip the problem down to its basics

Some problems are posed as idealized "toy model" problems, for example a point mass colliding with a uniform stick with negligible thickness. Other problems deal with more realistic setups that you might encounter in the real world, for example two skaters colliding and grabbing on to each other. When dealing with the latter type, the first thing you should do is strip the problem down to its basics. If possible, reduce the problem to point masses, sticks, massless strings, etc. Many real-life problems that look different at first glance end up being the same when reduced to the underlying toy model. So when you solve the toy-model version, you're actually solving a more general problem, which is a good thing.

Of course, you need to be careful that your toy model mimics the original setup correctly. For example, simplifying an object to a point mass works fine if you're using forces, but not necessarily if you're using torques. Your goal is to simplify the setup as much as possible without changing the physics. It takes some thought not to go too far, but this thought process is helpful in solving the problem. It helps you decide which aspects of the problem are important, and which aspects are irrelevant. This in turn helps you decide which physical principles you need to use (Strategy 10 below). Along these lines, if the original real-life setup is one for which you have some physical intuition, remember to use it when you start dealing with the toy model!

6. Choose wisely your coordinate system or reference frame

There are always only a couple of reasonable coordinate systems and reference frames to choose from, but a particular choice may greatly simplify things. For example, when dealing with an inclined plane, choosing tilted axes (parallel and perpendicular to the plane) is often helpful. And when dealing with circular motion, it is of course usually best to work with polar coordinates. And when two (or more) objects are moving with respect to each other, it is often helpful to analyze the setup in a new reference frame (the CM frame, or perhaps a frame moving along with one of the objects).

Part of choosing a coordinate system involves choosing the positive directions for the coordinate axes. This is completely your choice, but you must remember that once you pick a convention, you must stick with it. It's fine to let downward be positive for a falling object; just don't forget later on in your solution that you've made that choice.

7. Identify the initial and final states of the system

This is especially important in problems involving conservation principles (conservation of energy, momentum, angular momentum). In many cases, you can ignore the specifics of what happens during a process and simply equate the initial and final values of a particular quantity.

Another class of problems is "initial condition" problems. If you've calculated a general expression for, say, an object's position involving some unknown parameters, you can determine the values of these parameters by invoking the initial conditions (usually the initial position and velocity).

8. Identify the constraints

Is an object constrained to lie on a plane? Or travel in a circle? Or move with constant velocity? Is the system static? In the end, a constraint means that you have one fewer unknown than you otherwise might have thought. For example, if an object lies on a plane inclined at angle θ, then its coordinates are related by $y = x \tan\theta$. So if you choose x as your unknown, then y is determined.

9. Convert numbers to letters, so that you can solve things symbolically

This strategy is extremely helpful and very simple to apply. It is discussed in depth in Section 1.1.1 above, where its many benefits are noted.

1.2.2 Solving the problem

Having taken the above mechanical steps, it's now time to start thinking. There's no sure-fire way to guarantee that you'll solve every problem you encounter, but the following five strategies will certainly help.

10. Identify the physical principles involved

Think about what physical principle(s) will allow you to solve the problem. The most fundamental principles in mechanics are $F = ma$ and $\tau = I\alpha$ (or more accurately $F = dp/dt$ and $\tau = dL/dt$), and conservation of E, p, and L. A given problem can invariably be solved in multiple ways. For example, since conservation of energy can be derived from $F = ma$, any problem that can be solved with conservation of energy can also be solved with $F = ma$, although the latter solution may be more cumbersome.

In addition to the overarching fundamental principles listed above, there are many other physical principles/facts that you may need to use. For example, the radial acceleration is v^2/r (Eq. (3.7)); $v_y = 0$ at the highest point in projectile motion; the energy of an object that is both translating and rotating consists of two terms (Eq. (7.8)); Hooke's law for a spring is $F = -kx$ (Eq. (10.1)); Newton's law of gravitation is an inverse-square law (Eq. (11.1)); and so on.

11. Convert physical statements into mathematical equations

Having identified the relevant physical principles, you must now convert them into mathematical equations. For example, having noted that the horizontal speed in projectile motion is constant, you need to write down $x = (v_0 \cos\theta)t$, or something equivalent. Or having decided that you will use $F = ma$ to solve a problem, you need to explicitly write down the F_x (and maybe F_y and F_z) equations, which may involve breaking vectors into their components. Or having decided to use conservation of energy, you need to determine what kinds of energy are involved, and then equate the initial total energy with the final total energy.

12. Think initially in terms of physical statements, rather than equations

It is important to *first* think about the physical principles, and *then* think about how you can express them with equations. Don't just write down a bunch of equations and look for ways to plug things into them. *The initial goal when attacking a problem isn't to write down the correct equation; rather, it's to say the correct thing in words.* If you proceed by blindly writing down all the equations you can think of that seem somewhat relevant, you might end up just going around in circles. You wouldn't try to get to a certain destination by randomly walking around with the hope that you'll eventually stumble upon it. And that strategy doesn't work any better in problem solving!

13. Make sure you have as many facts/equations as unknowns

If you are trying to solve for three unknowns and you have only two equations/facts, then there's no way you're going to be successful. Along the same lines, if you've identified an unknown but haven't incorporated it into any of your equations/facts, then there's no way you're going to be able to solve for it. If you can't think of which additional physical principle to apply to generate the necessary equation, it's helpful to run through all of the given information and think about the implications of each bit.

14. Be organized

Sometimes you can see right away exactly how to solve a problem, in which case you can fly right through it, without much need for organization. But unless you're positive that the solution will be quick, it is critical to be organized about the other strategies in this list, by explicitly writing things out. For example, you should write out the knowns and unknowns, as opposed to just thinking them. And likewise for the physical principles involved, etc. There's no need to write a book, but some brief notes will do wonders in organizing your thoughts.

1.2.3 Troubleshooting

In many cases the preceding strategies are sufficient for solving a problem. But if you get stuck, the following thirteen strategies should be helpful.

The following three strategies are bread-and-butter ones.

15. Reduce the problem to an intermediate one

Equivalently, work backwards. Say to yourself, "I'd be able to get the answer to the problem if I somehow knew the quantity A. And I'd be able to get A if I somehow knew B." And so on. Eventually you'll hit a quantity that you can figure out from the given information. For example, you can find the distance x traveled by a projectile if you somehow know the time t (because $x = (v_0 \cos\theta)t$). So the problem reduces to finding t. And you can find t by (among other ways) noting that at the top of the projectile motion (after time $t/2$), the y component of the velocity is zero, so $v_0 \sin\theta - g(t/2) = 0$.

As an analogy, if you can't remember how to get to a certain destination, you're still in luck if you remember that it's just north of a park, which you remember is a few blocks down a certain street from a statue, which you remember is around the corner from a school, which you remember how to get to.

16. Exaggerate/change the parameters to understand their influence

This is basically the same as checking limiting cases of your final answer (Strategy 28 below, discussed in detail in Section 1.1.3). However, there is no need to wait until you obtain your final answer (or even an intermediate result) to take advantage of this extremely useful strategy. Your intuition about extreme cases is much better than your intuition about normal scenarios, so you

should use it. Once you see that a certain parameter influences the result, you can hone in on how exactly this influence comes about. This can then lead you to the relevant physical principle (Strategy 10).

17. Think about how the various quantities (known or unknown) are related

The task of Strategy 10 is to identify the relevant physical principles. This will yield relations among the various quantities. If you've missed some of the principles, it might be possible to figure out what they are by thinking about how the various quantities relate. For example, consider a mass on the end of a spring, and let's say you pull the mass a distance d away from its equilibrium position and then let go. It is intuitively clear that the larger d is, the larger the mass's speed v will be when it passes through the equilibrium position during the resulting oscillatory motion. If your goal is to find v, the preceding qualitative statement might help lead you to the useful physical principle of energy conservation, which will then allow you to write down a quantitative mathematical equation.

The following three strategies are quick checks.

18. Check that you have incorporated all of the given information

Part of the task of Strategy 2 is to identify everything that you know. When immersed in a problem, it's easy to forget some of this information, and this will likely make the problem unsolvable. So double check that for every given piece of information, you've either incorporated it or declared it to be irrelevant.

19. Check your math

Check over your algebra, of course. It's good to do at least a cursory check after each step. If you eventually hit a roadblock, go back and do a more careful check through all the steps.

20. Check the signs in all equations

In some sense this is just a subcase of the preceding strategy of checking your math. But often when people check through algebra, they fixate on the numerical values of the various terms and neglect the signs. So if you're stuck, just do a quick check where you ignore the numerical values and look only at the signs, just to make sure that at least those are correct. This check should be very quick. Pay special attention to the initial equation that you wrote down. A common mistake is to have an incorrect sign right from the start (for example, having the wrong sign in a vector component), which won't show up as an algebra mistake.

The following three strategies involve building on other knowledge.

21. Think of similar problems you know how to do

Try to reduce the problem (all, or part of it) to a previously solved problem. There are only so many types of problems in introductory mechanics, so odds are that if you've done a good number of problems, they should start looking familiar. How is the present problem similar to an old one, and how is it different?

You might wonder whether someone becomes an expert problem solver by being brilliant, or by solving a zillion problems, which has the effect of making any new problem look vaguely familiar. Elite athletes, chess players, debaters, comedians, etc., rely on recognizing familiar situations that they know how to react to. You can argue about what percentage of their strategy/action is based on this reaction. But you can't argue with the fact that a huge arsenal of familiar situations, built up from endless hours of practice, is a necessary condition for elite status in pretty much anything.

22. Think of a real-life setup that behaves the same way

This strategy in is the same spirit as the preceding one. We continually gather physical intuition from everyday life, so in a sense we're always practicing physics without even knowing it. For example, if you need to push a large object, say, a car, then you know that you should lean forward with your feet behind you, as opposed to pushing with your body upright. We have much more physical intuition about Newtonian mechanics than we do about other subfields of physics (electricity and magnetism, relativity, quantum mechanics), so you should use it to your advantage when solving mechanics problems!

Of course, there are times when your intuition might lead you astray, sometimes due to the fact that certain observations dominate others. For example, based on observations of skidding and general braking in a car or on a bike, you might think that friction always slows things down. It's easy to forget that when you accelerate from rest (often a more gentle acceleration than braking), friction is what speeds you up. You won't go anywhere if you're on ice!

23. Solve a simpler problem

If you can't get anywhere, if never hurts to solve a simpler version of the problem, to get a feel for what's going on. A particular example of this (which appears more often in math than in physics) is a problem that involves a large number or a general number N. In such problems, you definitely want to try solving things for the case of $N = 1, 2, 3$. Once you see what's going on for small numbers, it's much easier to generalize to an arbitrary number N.

The following four strategies involve helping your brain get going.

24. Explain (or imagine explaining) the solution to someone else

This strategy might seem a little silly, but it's really just a way of forcing yourself to organize your thoughts and proceed slowly. I assume I'm not the only person who has checked over an incorrect solution multiple times in my head, only to make the same mistake each time. It's very easy to repeatedly slide over a mistake or faulty assumption in the confines of one's own head. This can often be remedied by explaining your solution to someone else. And in most cases, you will see the mistake even before the other person says anything. But in the event that there's no one else around to lend an ear, a little talking to yourself never hurt.

25. Imagine your teacher explaining the solution

This strategy might also seem a little silly, but it does help sometimes, due to the fact that you're a human and not a machine. When floundering with a problem, it's easy to lose confidence and give up, even if you don't consciously know that you're giving up. If you imagine your teacher (or another student you look up to) explaining the solution, then since you expect them to be able to solve the problem with confidence, you just might end up solving it yourself. Occasionally it's better to ditch the "I think I can" mantra for "I *know* someone *else* can." Hey, if it gets the job done…

26. When you can't think of anything to do, just do something!

If all else fails, just start trying some random things. It can't hurt. You might hit something that gets you on track. A reasonable analogy is the unrealistic scenario in which you're lost in the woods with an infinite supply of food, and with zero chance that anyone else is going to help you. It doesn't do any good to just sit there. If you can't think of a reason to head in any particular direction, you should just head in *some* direction. Maybe you'll hit a stream that heads somewhere. Of course, after the fact, you'll probably see why you should have known the stream was there in the first place (a gulley between two hills, the dripping wet moose that walked past you, etc.). But that's knowledge you can use the next time you get stuck.

27. Set the problem aside and go for a walk

If nothing is working and you're truly at an impasse, take a break from the problem. It's easy to get stuck in a rut in your thinking, where you keep making the same mistakes and your mind keeps heading from a correct thought to an incorrect or unhelpful one. If you take a break and let your mind wander a little, it might semi-randomly jump to a helpful thought.

1.2.4 Finishing up

Assuming you've produced an answer to a problem, the following three strategies are ones you should always apply, as double checks on your answer.

28. Check limits and special cases

This strategy is immensely helpful. It is discussed in depth in Section 1.1.3, where its many benefits are explained.

29. Check units

This strategy is also very helpful and is discussed in depth in Section 1.1.2.

30. Check the rough size of numerical values

If your final answer involves actual numbers, be sure to check that they make sense. You can do this by checking that the rough "order of magnitude" (the nearest power of 10) is plausible. It's quite possible that you dropped a factor of 10 somewhere in the calculation. Or maybe you dropped one of the parameters when going from one line of the math to the next (although checking units is often a safeguard against this, too).

1.2.5 Looking ahead

After solving a problem, you can use the following four strategies to build upon what you've done and better prepare yourself for future problems.

31. Review the solution

After you're finished with a problem, go back and carefully review the entire solution. First think about the big-picture idea(s). Then think not only about what each step was, but also *why* you performed it. That is, how did each step logically follow from the previous one, as opposed to just being a random step? This review is extremely helpful in making the solution sink in, and it usually takes only a minute or two. You get a lot of bang for your buck, timewise.

Even if the solution *did* sink in first time around, there is another benefit to a careful review. It's often not enough just to know something; you also need to *know that you know it.* That way, you can confidently apply it to a future problem. If a certain tool is in your arsenal but you don't know that it's there, it doesn't help you much. (Think of all the memories that reside in your brain but that you'll never ever think of again, because you don't know that you have them.) A careful review of something will let you know that you know it.

32. Analyze where/why you went wrong

This strategy is actually relevant *while* you are solving the problem. As mentioned in the preface, it is critical that you never just read a solution straight through, unless you've already solved the problem. Just read enough to get a hint to get started. If you *do* need to read a little bit to get a hint, then before you use the hint to move on with the problem, think about your previous train of thought. Where exactly did you get stuck? What would you have needed to realize to

get unstuck? How will you modify your thinking in the future so that you don't hit the same roadblock?

In short, it doesn't hurt to obsess a little about where you went wrong and what you can do better. If you get schooled on the basketball court with a crossover dribble that leads to the game-winning layup, you're probably going to obsess for a while about what to do differently next time. That's a good thing. And it works the same way with problem solving. And for that matter, anything else you're trying to get good at.

33. Think of another approach

Just because you solved a problem once, that doesn't mean you can't solve it again. There are often many ways to solve a given problem, and you can learn a lot by working through another solution. Furthermore, most of what you learn actually comes from the things you do wrong, so if you happened to have breezed through your first solution, you might not have learned much. But if you work through a second solution and have to struggle here and there, you'll significantly increase what you take away.

34. Think of a variation or extension

Try to make up a similar problem. You could vary the parameters, of course. But if you've solved the problem symbolically instead of with numbers (as you should always do!), then you've actually already solved the problem for any values of the parameters. So there's nothing new there. So what can you change? Perhaps add new forces and/or objects, or change the shape of an object, or allow something that was fixed to be moveable, or change the direction of the motion, or relax some of the given information and see what the most general motion is, or generalize from 1-D to 2-D, etc. Thinking about variations will not only solidify the problem you just solved, but also make you much more prepared for new problems that come your way.

As mentioned at the start of this section, the above list of strategies is long, so you certainly shouldn't try to memorize it. Just refer back to it now and then.