2008 F=ma Exam: Problem 25

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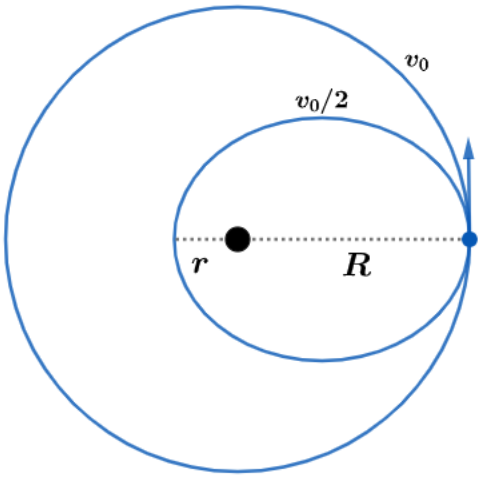

Recall for the circular orbit, we have

$$v_0 = \sqrt{\frac{GM}{R}}$$

For the elliptical orbit, the starting point is the apogee so

$$R = a + c$$
$$\frac{1}{2}v_0 = v_{\text{apogee}} = \frac{a-c}{b}\sqrt{\frac{GM}{a}} = \sqrt{\frac{a-c}{a+c}}\sqrt{\frac{GM}{a}}$$

using our results for elliptical orbits (note $b^2 = a^2 - c^2$). Substituting these into the first equation,

$$2\sqrt{\frac{a-c}{a+c}}\sqrt{\frac{GM}{a}} = \sqrt{\frac{GM}{a+c}}$$
$$2\sqrt{1-\frac{c}{a}} = 1$$
$$\frac{c}{a} = \frac{3}{4}$$

We have

$$\frac{r}{R} = \frac{a-c}{a+c} = \frac{1-(c/a)}{1+(c/a)} = \frac{1/4}{7/4} = \frac{1}{7}$$

so the answer is $\boxed{\text{E}}$.